# Static Liquid Holdup in Packed Beds of Spherical Particles


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The static liquid holdup in a packed bed is defined as the volume fraction of liquid, referred to total bed volume, that remains in the bed after complete draining. The liquid that remains in the bed is prevented from draining by a balance between gravitational and surface tension forces.

The static liquid holdup is commonly used as a parameter in the characterization of the hydrodynamics of gas-liquid flow through packed beds. Shulman et al. (1955) indicated that, under gas-liquid flow conditions, the liquid holdup in a packed bed could be visualized as made up of static and dynamic contributions. Afterwards, several investigators have used this idea in the development of empirical correlations for calculating pressure drops and liquid holdups in trickling flow through packed beds. In this line of thought, Specchia and Baldi (1977) correlated the two-phase pressure drop through the bed in terms of an Ergun-type equation whose characteristic constants were evaluated in terms of the pressure drop through the "wetted packing," that is, when the liquid present was that corresponding to the static holdup. The liquid holdup that is usually correlated to the gas and liquid flow rates is usually the dynamic holdup (Specchia and Baldi, 1977; Sáez and Carbonell, 1985). This is a very convenient representation since it can be easily forced to yield a zero value for the case of no flow.

In the literature, the static liquid holdup has been empirically correlated as a function of the Eötvös number:

$$E\ddot{o} = \frac{\rho g l^2}{\sigma} \tag{1}$$

where $\rho$ is the density of the liquid phase, $g$ is the acceleration of gravity, $\sigma$ is the gas-liquid interfacial tension, and $l$ is a characteristic length that represents the geometry of the packing particles. The choice of characteristic length differs in the available correlations. Charpentier et al. (1968) proposed that $l$ be chosen as the nominal particle diameter, $l = d$. Sáez and Carbonell (1985) selected a parameter proportional to the hydraulic diameter as the characteristic length:

$$l = \frac{d_e \, \epsilon}{1-\epsilon} \tag{2}$$

where $\epsilon$ is the porosity of the packed bed and $d_e$ is the equivalent particle diameter, defined as the diameter of the sphere that has the same volume-to-surface ratio of the particle. Dombrowski and Brownell (1954) used a characteristic length proportional to the square root of the absolute permeability of the bed.

Turner and Hewitt (1959) developed a correlation based on the experimental measures of liquid volumes held at menisci between spheres. Their results, however, were limited to large particles (with diameters over 1 cm), which correspond to Eötvös numbers larger than the usual range encountered in packed beds. Recently, Reddy et al. (1990) used the experimental data reported by Turner and Hewitt to propose a model with the objective of exploring the distribution of liquid flow in trickle-bed reactors. The model used a geometric variable related to the geometry of the meniscus formed between the particles as an adjustable parameter.

The static liquid holdup is also relevant in the modeling of mixing and transport processes in gas-liquid, flowthrough packed beds, in which an amount of liquid equal to the static liquid holdup is commonly considered as a stagnant region under gas-liquid flow conditions. The dynamic response of perturbations in tracer concentrations has been used to determine the static holdup from dispersion experiments (Bennett and Goodridge, 1970), although recent results suggest that the static holdup obtained from such methods differs appreciably from that obtained from draining experiments (Schubert et al., 1986).

In this work, we use dimensional analysis to establish the independent parameters that affect the static liquid holdup. We also present experimental data to explore the dependence of the static holdup on the Eötvös number; through comparison with previous theoretical works, we analyze the mechanisms responsible for the effect of Eötvös number on the static liquid holdup.

## Experimental Study

The static liquid holdup was measured in this work for three


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**Table 1. Physical Properties of Liquids**

| Liquids | $\rho$ (kg/m$^3$) | $\sigma$ (mN/m) |
|---|---|---|
| Water | 1,000 | 73.1 |
| Methanol | 792 | 22.7 |
| Kerosene | 808 | 25.0 |

different liquids (Table 1) and packings of spherical glass beads with narrow size distributions of diameters ranging from 0.5 to 4.0 mm. The experiments were performed at atmospheric pressure and 20°C in a cylindrical packed bed with a diameter of 7.5 cm and a packing height of 50 cm. Before the experiments, the glass beads were carefully cleaned with an acid solution to ensure perfect wetting properties.

The volume of liquid required to fill the packing section of the bed ($V_h$) can be used to calculate the porosity of the medium, $\epsilon = V_h/V$, where $V$ is the total volume of packing section. In all the packings employed, the porosity was always with the range 0.35–0.39. After the bed was completely filled with liquid, it was allowed to drain for 60 minutes by opening a valve located at the bottom of the column. This period of time was determined to be more than enough to ensure a complete draining for the conditions employed in this work. The volume of liquid drained ($V_d$) was then used to calculate the static liquid holdup, $\epsilon_s = (V_h - V_d)/V$. Each experiment was repeated ten times to assess the reproducibility of the measures. We found that the maximum difference between each value of the drained volume and its mean over the ten experiments rarely exceeded 5%.

In the draining process, two regimes of flow are observed. The first period of draining is observed starting from the bed completely filled with liquid, in which most of the liquid drains forming a front that displaces downward. It is followed by the second period, in which the liquid flows through liquid films that form over the particles. The second period ends when the films break over the particles, leaving the static liquid holdup in the bed as menisci formed between particles. In this work, we analyzed the effect of the draining rate on the static liquid holdup by performing experiments with the same systems at different draining rates. The draining rate was controlled by means of the valve located at the bottom of the bed. We found, after extensive experimentation, that the differences observed between static liquid holdups at different draining rates were within the reproducibility limit. This indicates that the draining rate has no effect on the static liquid holdup. This result is due to the fact that the draining rate controls the motion of the front of liquid in the first period of draining, but has no apparent effect on the second period, in which the flow of liquid through the films is determined exclusively by the gravitational pull. It is worthwhile to mention that Turner and Hewitt (1959) reported a dependence of the static holdup on the draining rate for beds of large particles. This indicates that the existence of a dynamic effect was not observed in the experiments performed in this work.

## Analysis and Results

Consider the distribution of a liquid phase under static conditions in an unconsolidated porous medium. The main purpose of the following discussion is to establish, through dimensional analysis, the parameters that affect the liquid distribution.

The pressure distribution in the liquid phase under static conditions is given by:

$$\nabla P_l = \rho g \quad \text{in } V_l \tag{3}$$

where $V_l$ represents the liquid phase, and $g$ is the acceleration of gravity vector. The boundary condition at the gas-liquid interface ($A_{gl}$) is the equation of Young-LaPlace:

$$P_g - P_l = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad \text{at } A_{gl} \tag{4}$$

where the pressure of the gas phase ($P_g$) will be considered uniform; $r_1$ and $r_2$ are the local principal radii of curvature of the gas-liquid interface, which can be related to the shape of the interface.

An additional condition can be imposed at the contact line formed by the intersection between the gas-liquid interface and the gas-solid interface ($C$). Under static conditions, it is appropriate to use the contact angle ($\phi$) as a geometric constraint on the shape of the gas-liquid interface at the contact line as follows:

$$\mathbf{n}_s \cdot \mathbf{n}_f = \cos\phi \quad \text{at } C \tag{5}$$

where $\mathbf{n}_s$ and $\mathbf{n}_f$ are unit vectors normal to the gas-solid interface and the gas-liquid interface, respectively, pointing into the gas phase.

In addition, to specify completely the liquid pressure field, the value of $P_l$ should be specified at a point within $V_l$:

$$P_l = P_{l,0} \quad \text{at a point in } V_l \tag{6}$$

Consider the following dimensionless variables,

$$P^* = \frac{P_l - P_g}{\rho g l} \tag{7}$$

$$r_i^* = \frac{r_i}{l}; \quad \nabla^* = l\nabla \tag{8}$$

Equations 3, 4 and 6 can be expressed in dimensionless form as follows:

$$\nabla^* P^* = \mathbf{u}_g \text{ in } V_l \tag{9}$$

$$P^* = -\frac{1}{E\ddot{o}}\left(\frac{1}{r_1^*} + \frac{1}{r_2^*}\right) \quad \text{at } A_{gl} \tag{10}$$

$$P^* = P_0^* \quad \text{at a point in } V_l \tag{11}$$

where $\mathbf{u}_g$ *is a unit vector parallel to g.*

Once the local geometry of the solid phase (that is, the location of the solid surface) is known and a value of $P_0^*$ is specified, Eqs. 9 to 11 and Eq. 5 can be solved to find the shape and location of the gas-liquid interface ($A_{gl}$). The volume of liquid can be determined by integration and the liquid holdup can then be evaluated. The formulation presented indicates *that the liquid holdup in the medium is governed by the following relation:*

$$\epsilon_l = \epsilon_l\ (P_0^*,\ E\ddot{o},\ \phi,\ \text{local geometry}) \qquad (12)$$

The procedure outlined above has been applied in previous works to determine the shape of menisci formed at the contact point between parallel cylinders (Sáez and Carbonell, 1987) and vertically aligned spheres (Sáez and Carbonell, 1990). For specified values of $E\ddot{o}$ and $\phi$ and a given local geometry, Eq. 12 indicates that the liquid holdup depends on the dimensionless datum pressure, $P_0^*$. This means that there are many physically feasible menisci. There is, however, a limit to the size of the meniscus that can be attained. In some cases, as the size of the meniscus increases, a point is reached at which the meniscus becomes unstable. For aligned cylinders (Sáez and Carbonell, 1987), the stability condition yields a value of $P_0^*$ below which no stable menisci can be formed. In other cases, stable menisci with relatively large volumes can be formed, but there is a geometrical constraint owing to the shape of the particles that does not allow the formation of larger menisci. This geometrical limit is also related to a value of $P_0^*$ below which menisci cannot be physically feasible. This condition yields menisci of maximum possible volume for the case of touching spheres (Sáez and Carbonell, 1990).

At the end of the draining process from a packed bed, it seems reasonable to assume that when the breakage of the liquid films occurs, the amount of liquid that remains in the resulting menisci is the possible maximum amount that can reach a stable configuration. From this point of view, the static liquid holdup in a packed bed can be conceived as the value obtained from Eq. 12 for the limiting value of $P_0^*$ previously described. The resulting relation can then be expressed as follows:

$$\epsilon_s = \epsilon_s\ (E\ddot{o},\ \phi,\ \text{local geometry}) \qquad (13)$$

Equation 13 indicates explicitly that the static liquid holdup should be affected only by the Eötvös number, the contact angle, and the local geometry of the solid surface. For the case of parallel cylinders and menisci between vertically aligned spheres, the representation given by Eq. 13 has been rigorously evaluated in previous works (Sáez and Carbonell, 1987, 1990). The large differences observed between those two simple geometries for the same values of $E\ddot{o}$ and $\phi$ suggest that the local geometry has a strong effect on the static holdup. This reflects the fact that the meniscus of maximum volume is very sensitive to the geometrical constraint imposed by the geometry of the solid particles. In addition, the effect of the contact angle is appreciable, and it does not exhibit monotonous trends: depending on the values of $\phi$ and $E\ddot{o}$, an increase in the contact angle might result in a decrease or an increase in the static holdup.

For real packed beds, finding the relation expressed by Eq. 13 seems to be a formidable problem at this point, owing to the high complexity of the local geometry. Empirical relations such as those proposed by Charpentier et al. (1968) and Sáez and Carbonell (1985) appear to be the only realistic way to assess the quantitative description of static holdup. Those correlations, however, have been developed by using experimental data reported in the literature that correspond to packings of different shapes, thus altering the local geometry of the solid surface. The use of various characteristic lengths in the correlations has not been enough to characterize the differences in local geometry exhibited by the packings used. Besides, the contact angle has not been reported in the experiments, so that the data used might correspond to various values of this parameter. These two facts might explain the large scatter observed in the comparison of the correlations with the experimental data.

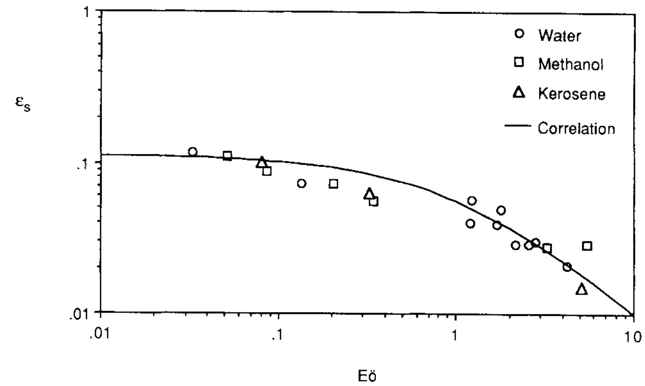

**Figure 1. Static liquid holdup: experimental data**

In the experiments performed in this work, we used randomly-packed beds of spherical particles, and possible impurities at the solid surface (glass) were removed to ensure that the contact angle is close to $\phi = 0$. Under these controlled conditions, Eq. 13 indicates a direct relation between the static holdup and the Eötvös number. Figure 1 shows the experimental data gathered in this work for three different liquids and various particle diameters. The plot suggests a unique dependence between $\epsilon_s$ and $E\ddot{o}$, regardless of the liquid used. The data are well fitted by the following equation, which is also plotted in Figure 1,

$$\epsilon_s = \frac{0.11}{1 + E\ddot{o}} \qquad (14)$$

When Figure 1 is compared with the correlations developed in the literature (see, for example, Charpentier et al., 1968; Sáez and Carbonell, 1985; Wammes et al., 1990), several important observations can be made.

First of all, the observed scatter of the data is appreciably smaller in Figure 1, which is a result of the controlled conditions used in this work. The fact that liquids of different viscosities are in agreement with a single representation is consistent with the observation made previously regarding the absence of dynamic effects on the static holdup.

Second, the experimental data obtained in this work suggest a limiting value of $\epsilon_s = 0.11$ for very small $E\ddot{o}$ (when gravitational effects become negligible), whereas a value of 0.05 has been traditionally reported to be the maximum static holdup (Charpentier et al., 1968; Wammes et al., 1990). This difference is a result of the fact that we have reached smaller Eötvös numbers than those reported in the literature (reported data fall in the range $E\ddot{o} > 0.3$). It is important to point out that, if the particles are small enough, a point is reached for which the liquid retained in the bed is not the product of pendular menisci formed at the contact point between the particles, but a consequence of the formation of liquid blobs that engulf several particles. Essentially, a static holdup equal to the porosity can be obtained by this mechanism of liquid retention.

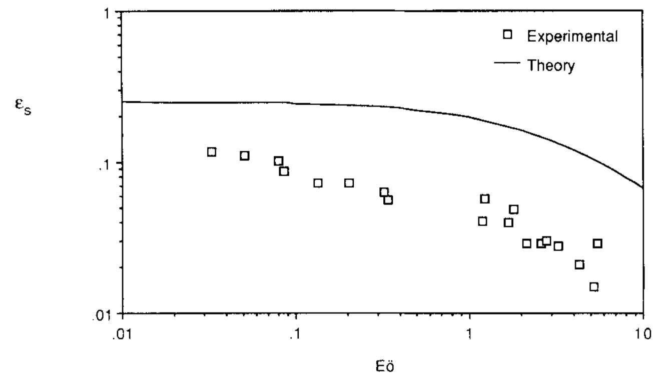


**Figure 2. Static holdups: randomly-packed beds of spheres vs. vertically-aligned touching spheres.**

Our analysis is restricted to particles large enough that the latter mechanism is not the controlling one. Even with this fact in mind, our work indicates that static holdups larger than 0.05 can be obtained by menisci retention.

Third, our static holdup data are more sensitive to the Eötvös number than those previously reported. For instance, Charpentier et al.'s representation suggest a value of the static holdup around 0.3 for $E\ddot{o} = 10$, whereas our measurements indicate appreciably lower values (Figure 1). This difference might be a result of the effects of local geometry and contact angle.

In Figure 2 we compare all the experimental data obtained in this work with relation 13 as obtained theoretically by Sáez and Carbonell (1990) for menisci between vertically aligned spheres for $\phi = 0$. It can be seen that the theory greatly overestimates the experimental data. This is a consequence of the difference in local geometry between both situations. Note that, in real packed beds, the touching point between any couple of spheres is not vertically aligned, as in the case of the theory. However, the representation in Fig. 2 shows that both the theory and the experimental data exhibit the same trend with regards to variations in the Eötvös numbers. This fact leads us to think that the mechanism responsible for the static liquid holdup in both cases is similar: the static holdup is a consequence of the destabilizing effect of the gravitational force, combined with the influence of the local geometry of the packing particles.

The results suggest that a general empirical relation for calculating the static liquid holdup in packed beds should be established based on the representation in Eq. 13. The other two parameters that have not been usually taken into account (contact angle and local geometry) can have a very strong influence on the static holdup and, therefore, should not be disregarded.

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